Honors Calculus
Summer Preparation 2021

*Name:*______________________________ *ARCHBISHOP CURLEY HIGH SCHOOL*

Honors Calculus

Summer Work and List of Topical Understandings

In order to be a successful student in the Honors Calculus course, it is imperative to practice and improve your current math skills. Calculus is a course in advanced algebra and geometry that is used to study how things change. To be successful in this course, students must be highly motivated, dedicated, capable learners who put forth great effort in working independently to fully understand the concepts in the honors calculus curriculum. The summer packet will act as a refresher of math concepts that should already be mastered when you enter the honors calculus course in September. This packet is a **requirement** for students entering honors calculus and is due the first week of class. Complete as much of this packet on your own as you can, then get together with a friend, e-mail me, or "google" the topic. As always, your best work is expected!

Requirements for completing summer packet:

- ✓ You must complete each of the problems in the packet.
- ✓ You must show all of your work in the space provided on the packet or on separate paper, attached to the packet and labelled on each page.
- ✓ Be sure all problems are neatly organized and all writing is legible.
- ✓ In the event that you are unsure how to perform functions on your calculator, you may need to read through your calculator manual to understand the necessary syntax or keystrokes. You must be familiar with certain built-in calculator functions such as finding values, intersection points, using tables, and zeros of a function.
- ✓ I expect you to come in with certain understandings that are prerequisite to Calculus. A list of these topical understandings is below.

Summer Work Topics:

- ❖ Factoring
- ❖ Zeros/roots/x-intercepts of rational and polynomial functions
- ❖ Polynomial Long Division
- ❖ Completing the square
- ❖ Write the equation of a line
- ❖ Quadratic formula
- ❖ Unit Circle
- ❖ Composite function and notation
- ❖ Solving trigonometric equations
- ❖ Domain/Range
- ❖ Interpreting and comprehending word problems
- ❖ Graphing, simplifying expressions, and solving equations of the following types:
  - ❖ Trigonometric
  - ❖ Rational
  - ❖ Piecewise
  - ❖ Logarithmic
  - ❖ Exponential
  - ❖ Polynomial
  - ❖ Power
  - ❖ Radical

Topic 1: Properties of Exponents/Radicals

PROPERTY		EXAMPLE
Product of Powers	$a^m \bullet a^n = a^{m+n}$	$x^4 \bullet x^2 = x^6$
Power of a Power	$(a^m)^n = a^{m \bullet n}$	$(x^4)^2 = x^8$
Power of a Product	$(ab)^m = a^m b^m$	$(2x)^3 = 8x^3$
Negative Power	$a^{-n} = \dfrac{1}{a^n}$ $(a \neq 0)$	$x^{-3} = \dfrac{1}{x^3}$
Zero Power	$a^0 = 1$ $(a \neq 0)$	$4^0 = 1$
Quotient of Powers	$\dfrac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$	$\dfrac{x^3}{x^2} = x^1 = x$
Power of Quotient	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$ $(b \neq 0)$	$\left(\dfrac{x}{y}\right)^3 = \dfrac{x^3}{y^3}$
Fractional Exponents	$x^{\frac{1}{2}} = \sqrt{x}$ $x^{\frac{2}{3}} = \sqrt[3]{x^2}$	$9^{\frac{1}{2}} = \sqrt{9} = \pm 3$ $8^{\frac{2}{3}} = \sqrt[3]{8^2} = 4$
Radical Products	$\sqrt{a} \bullet \sqrt{b} = \sqrt{ab}$	$\sqrt{2} \bullet \sqrt{3} = \sqrt{6}$
Radical Quotients	$\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\dfrac{25}{9}} = \dfrac{\sqrt{25}}{\sqrt{9}} = \dfrac{5}{3}$
Rationalizing radical denominators	$\dfrac{a}{\sqrt{b}} \bullet \dfrac{\sqrt{b}}{\sqrt{b}} = \dfrac{a\sqrt{b}}{b}$ $\dfrac{a}{1+\sqrt{b}} \bullet \dfrac{1-\sqrt{b}}{1-\sqrt{b}} = \dfrac{a-a\sqrt{b}}{1-b}$	

Simplify:

1) $g^5 \bullet g^{11}$ ________

2) $(b^6)^3$ ________

3) w^{-7} ________

4) $\dfrac{y^{12}}{y^8}$ ________

5) $(3x^7)(-5x^{-3})$ ________

6) $(-4a^{-5}b^0c)^2$ ________

7) $\dfrac{-15x^7y^{-2}}{25x^{-9}y^5}$ ________

8) $\left(\dfrac{4x^9}{12x^4}\right)^3$ ________

Express the following in simplest radical form.

9) $\sqrt{50}$

10) $\sqrt{24}$

11) $\sqrt{192}$

12) $\sqrt{169}$

13) Simplify $\dfrac{\sqrt[7]{x^9}}{\sqrt[5]{x^6}}$. Express your answer using a single radical.

Topic 1: Simplifying Complex Fractions

When simplifying complex fractions, you want to eliminate the "little denominators" by multiplying through what would be the LCM (Least Common Multiple) of them. You end up multiplying the "big numerator" and the "big denominator" by that LCM.

For example:

1) $\dfrac{1+\frac{1}{a}}{\frac{2}{a^2}-1}$ The LCM of the "little denominator" would be a^2. So multiply the entire big numerator as well as the entire "big denominator" by a^2. Don't forget to distribute properly!

$$\frac{\left(1+\frac{1}{a}\right)a^2}{\left(\frac{2}{a^2}-1\right)a^2}=\frac{a^2+a}{2-a^2}$$

2) $\dfrac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}$ The denominator you want to eliminate is simply $x+1$… so that's what you should multiply through by. Again, don't forget to distribute in the "big numerator" properly!

$$\frac{\left(-7-\frac{6}{x+1}\right)x+1}{\left(\frac{5}{x+1}\right)x+1}=\frac{-7x-7-6}{5}=\frac{-7x-13}{5}$$

3) $\dfrac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}}$ The LCM of the "little denominator" would be $x(x-4)$ So multiply the entire "big numerator" as well as the entire "big denominator" by $x(x-4)$. Don't forget to distribute properly!

$$\frac{\left(\frac{-2}{x}+\frac{3x}{x-4}\right)x(x-4)}{\left(5-\frac{1}{x-4}\right)x(x-4)}=\frac{-2x+8+3x^2}{5x^2-20x-x}=\frac{3x^2-2x+8}{5x^2-21x}$$

Simplify:

14) $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}=$

15) $\dfrac{1-\frac{1}{x^3}}{3+\frac{1}{x^2}}=$

16) $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}=$

17) $\dfrac{5+\frac{1}{n}-\frac{6}{n^2}}{\frac{2}{n}-\frac{2}{n^2}}=$

Topic 2: Complete Factorization of Polynomials

Factorization of polynomials is one of those topics that shows up throughout AP Calculus. You should be able to do it "on demand" and as the need arises.

Factoring Patterns:

Differences of Two Squares	$a^2 - b^2 = (a+b)(a-b)$
Sum of Two Cubes	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
Difference of Two Cubes	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Factor completely:

18) $5x^2 - 32x - 21$

19) $4x^2 + 20x + 9$

20) $15x^3 - 25x^2 + 75x - 125$

21) $x^2 + 15x + 56$

22) $28x^3 - 7x$

23) $216x^3 + 1$

Topic 3A: Linear Equations and Inequalities

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: $y = c$ (slope is 0)

Standard form: $Ax + By = C$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

24) Write an equation of a line with slope 3 and y-intercept 5.

25) Use point-slope form of a linear equation to find an equation of the line passing through the point (6, 5) with a slope of 2/3.

26) Find an equation of a line passing through the points (-3, 6) and (1, 2).

27 Find an equation of a line with an x-intercept of (2, 0) and a y-intercept of (0, 3).

Topic 3B: Quadratic Equations and Inequalities

To find the vertex from a ***quadratic equation in standard form:***

$y = ax^2 + bx + c$ We use the formula: $x = \frac{-b}{2a}$ Vertex at: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

If a is > 0 then opens upward if a is < 0 then opens downward

Quadratic equation in vertex form: $\boldsymbol{f(x) = a(x-h)^2 + k}$ Vertex at (h, k)

Quadratic Formula

y = (x - root$_1$) (x - root$_2$) x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

28. Find an equation for the parabola whose vertex is $(2,-5)$ and passes through $(4,7)$. Express your answer in the vertex form for a quadratic.

29. Solve the inequality: $x^2 - x - 12 > 0$.

A. $(-\infty,-4)\cup(3,\infty)$ B. $x=4,\ x=-3$

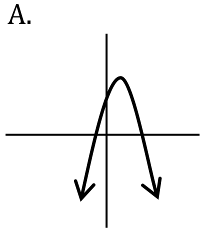
C. $(-3,4)$ D. $(-\infty,-3)\cup(4,\infty)$

30) Transform y = -3x^2 – 24x + 11 to vertex form by completing the square.

Topic 4: Polynomial Functions

31. Which of the following could represent a complete graph of $f(x) = ax - x^3$, where a is a real number?

A.



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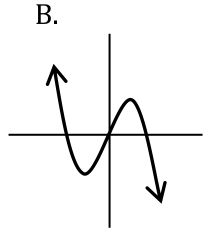

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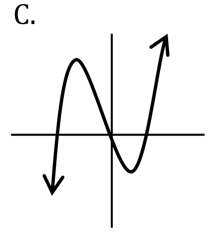

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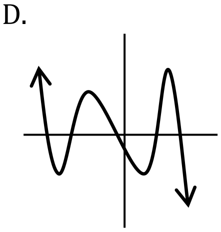


32) Find a degree 3 polynomial with zeros -2, 1, and 5 and going through the point $(0,-3)$.

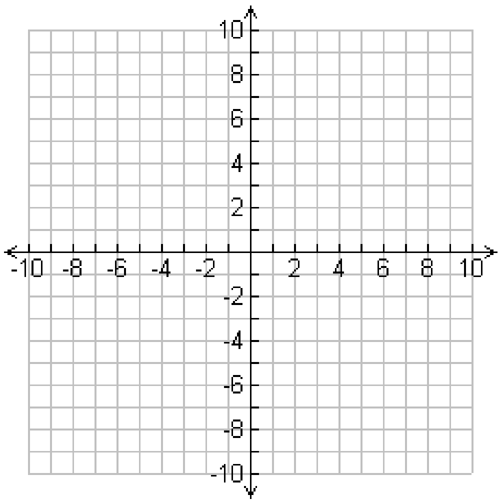
33) Use polynomial long division to rewrite the expression $\dfrac{x^3 + 7x^2 + 14x - 8}{x-4}$

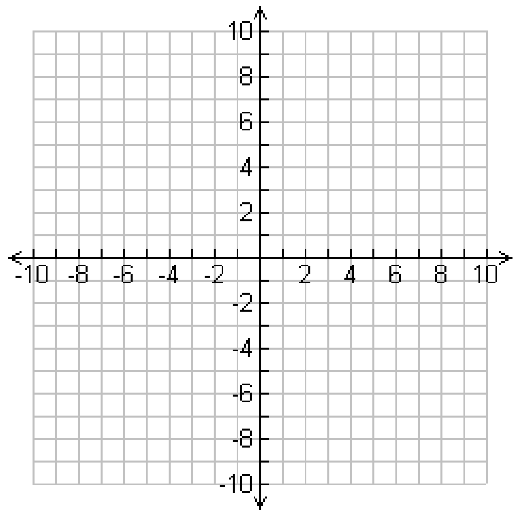
34) Use a graphing calculator to approximate all of the function's real zeros. Round your results to four decimal places. $f(x) = 3x^6 - 5x^5 - 4x^3 + x^2 + x + 1$

Topic 5: Graphs of Parent Functions

The following functions are the basic "parent" functions that you should know instantly. Actual points should be plotted (at least three) in the creation of these graphs. You should be able to produce these "on demand" and know them "in your head" without having to graph them on your GC.

35) Graph each function:



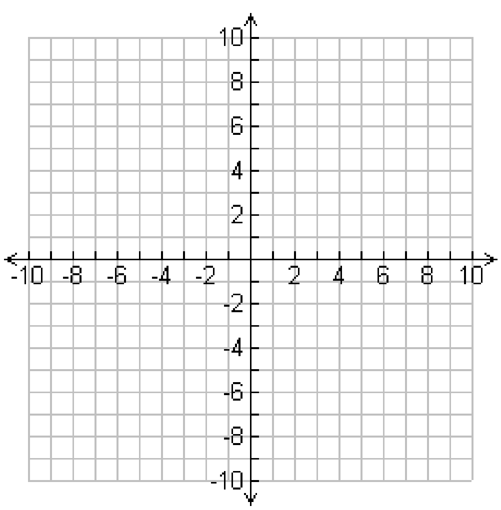


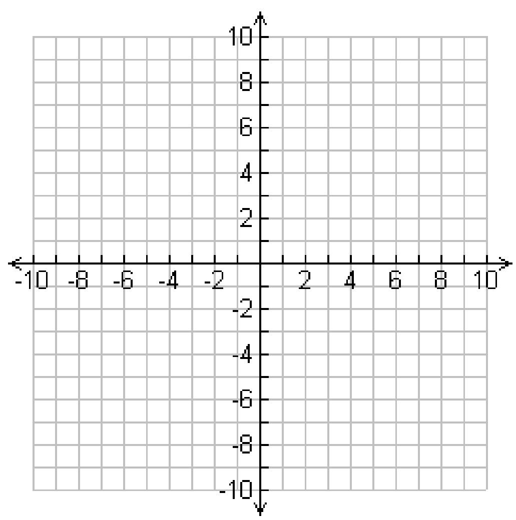
$f(x) = x$

Domain Range

$f(x) = |x|$

Domain Range



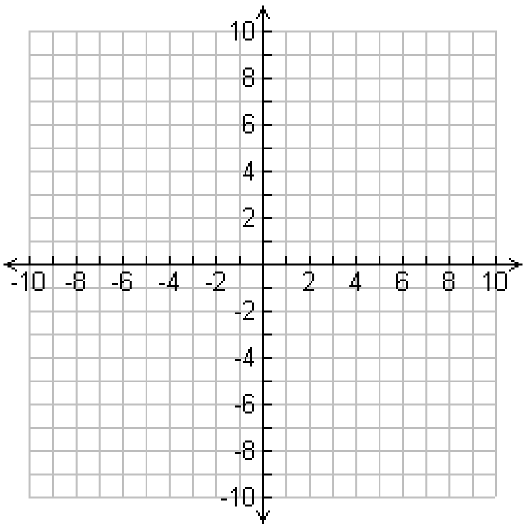


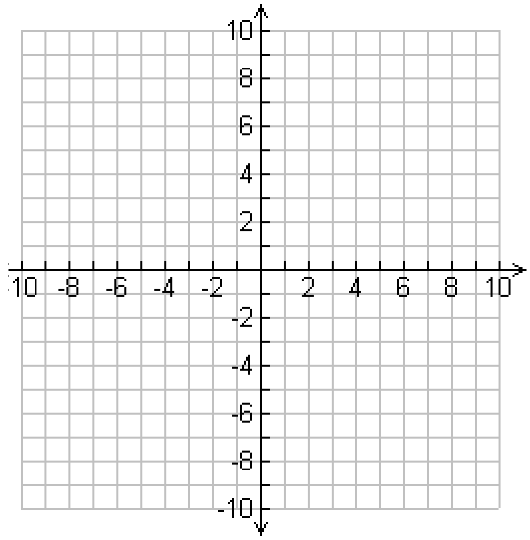
$f(x) = x^2$

Domain Range

$f(x) = \sqrt{x}$

Domain Range



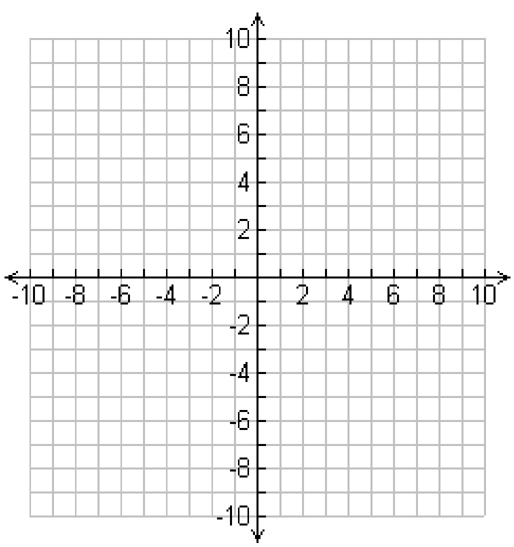


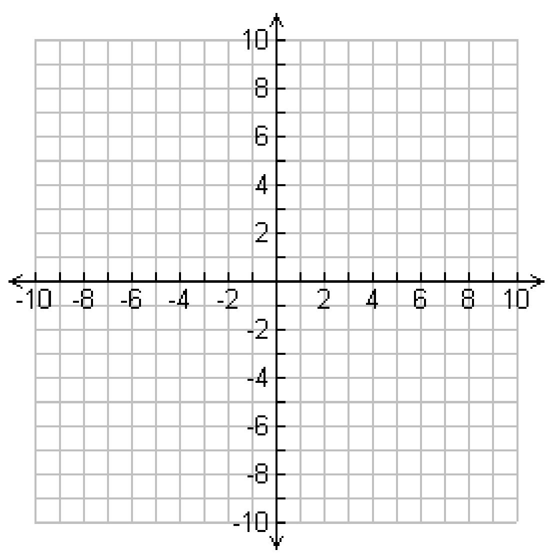
$f(x) = x^3$

Domain Range

$f(x) = \sqrt[3]{x}$

Domain Range





$f(x) = 2^x$

Domain Range

$f(x) = logx$

Domain Range

Topic 6: Rational Functions and Asymptotes

A rational function is a function of the form:

$$R(x)=\frac{p(x)}{q(x)}$$

Where *p* and *q* are functions

The Domain of a Rational Function

We need to make sure that the *denominator* is not equal to zero.

$$R(x)=\frac{p(x)}{q(x)}$$

$q(x)\neq 0$

Examples:

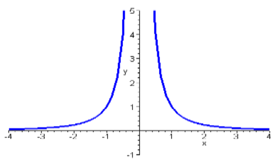
$R(x)=\frac{5x^2}{3+x}$ $\quad x\neq -3$

$H(x)=\frac{x-3}{(x+2)(x-2)}$ $\quad x\neq -2, 2$

$F(x)=\frac{x-1}{x^2+5x+4}$ $\quad x\neq -4, -1$

The graph looks like:

$f(x)=\frac{1}{x^2}$



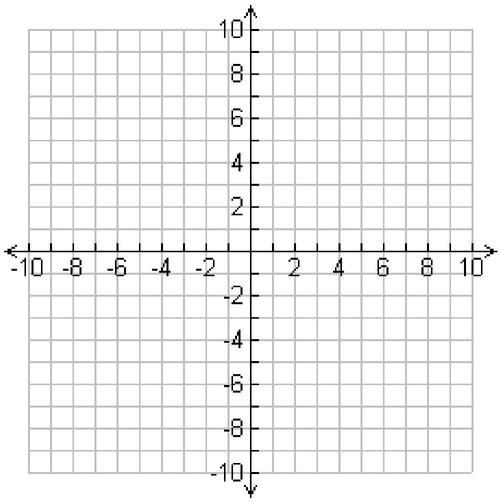
Since the graph is never equal to zero, the graph gets closer and closer to x = 0 but never touches the y-axis.

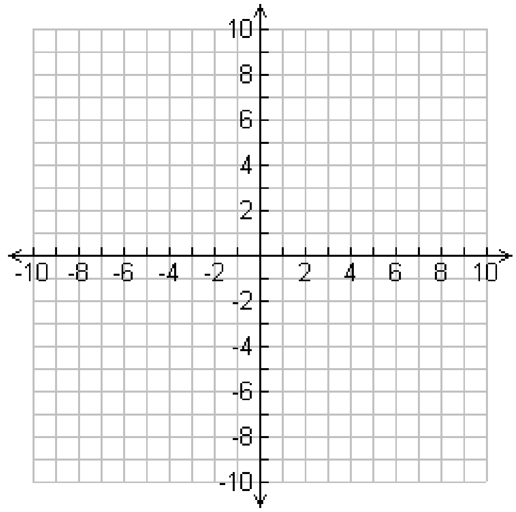
This line is called a *vertical asymptote*

We compare the *degree* in the numerator and in the denominator to tell us about *horizontal asymptotes.*

- If the degree of the numerator is <u>*less than*</u> the degree of the denominator, $R(x)=\frac{2x+5}{x^2-3x+4}$
  - *y = 0* is a horizontal asymptote.
- If the degree of the numerator is to the degree of the denominator, then there is a horizontal asymptote at the line $y=\frac{coefficient\ of\ lead\ term\ of\ numerator}{coefficient\ of\ lead\ term\ of\ denominator}$
- ***Example:*** $R(x)=\frac{2x^2+4x+5}{x^2-3x+4}=\frac{2}{1}=2$ So there is a horizontal asymptote at $y = 2$
- *Oblique Asymptote:*
- If the degree of the numerator is <u>greater than</u> the degree of the denominator, then there is no horizontal asymptote, but an oblique (slant) one. The equation is found by doing long division and the quotient is the equation of the oblique asymptote ignoring the remainder.
- ***Example:*** $R(x)=\frac{x^3+2x^2-3x+5}{x^2-3x+4}=(x+5)remainder\ \frac{(8x+5)}{(x^2-3x+4)}$ Slant asymptote is at ***y = x+5***

36) Graph the following rational functions. Label all asymptotes and/or holes in the graph. Identify the key points of the graph.





$f(x) = \frac{1}{x}$

Domain
Intercepts
Vertical Asymptotes
Horizontal Asymptotes

$f(x) = \frac{2x^2}{x^2-4}$

Domain
Intercepts
Vertical Asymptotes
Horizontal Asymptotes

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37) Give that $f(x) = \dfrac{\sqrt{x+5}}{x+2}$. Find the asymptotes and the domain of the function.

Domain: ___________

Vertical Asymptote(s): __________

Horizontal Asymptote(s): _________

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38) Factor to solve the inequality. Write your answer in interval notation. $0 \le \dfrac{x^3-64}{x-3}$

Topic 7: Function Composition and Inverses

There are two notations for the composition of functions: $f[g(x)]$ or $(f \circ g)(x)$. Each is read as "*f* of *g* of *x*" or "*f* compose *g* of *x*". Functions are composed by substituting the second function in place of the variable in the first function.

For example, if $f(x) = 2x^2 + 1$ and $g(x) = x - 4$.

$$f(g(x)) = f(x-4) = 2(x-4)^2 + 1 = 2(x^2 - 8x + 16) + 1 = 2x^2 - 16x + 33$$

39) Let $f(x) = \sqrt{x-3}$ and $g(x) = x^2 + 1$. Compute $(g \circ f)(x)$,

40) Find f(g(x)) if $f(x) = 2x^2$ and $g(x) = x + 4$

To find the inverse $f^{-1}(x)$ of a function $f(x)$, switch the *x* and *y*, then solve the new equation for *y*.

For example, find the inverse of $f(x) = \sqrt[3]{x+1}$.

$$y = \sqrt[3]{x+1}$$

$$x = \sqrt[3]{y+1}$$

$$x^3 = y + 1$$

$$f^{-1}(x) = x^3 - 1$$

41) Let $f(x) = \dfrac{3x+7}{x-2}$. Find $f^{-1}(x)$, the inverse of $f(x)$.

42) Find $f^{-1}(x)$ if $f(x) = \sqrt{x^2 + 16}$

Topic 8: Logarithmic and Exponential Functions

Evaluating Logarithms:

Examples:

- $\log_2 8 = 3$ because $2^3 = 8$
- $\log_{16} 4 = \frac{1}{2}$ because $16^{1/2} = \sqrt{16} = 4$
- $\log_2\left(\frac{1}{8}\right) = -3$ because $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
- $\ln e^3 = 3$ because a Natural Logarithm is just a logarithm in base e.

The Laws of Logarithms:

For all $a > 0$ $(a \neq 1), n \in R$ (is a real number), and $u, v \in R^+$ (u and v are positive real numbers)

1. $\log_a(uv) = \log_a u + \log_a v$
2. $\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$
3. $\log_a u^n = n \log_a u$
4. $\log_a u = \log_a v$ if and only if $u = v$

Exponential Functions:

$f(x) = a \bullet b^x$ where ***b*** is the base and ***x*** is the exponent.

If the bases are equal, then $b^{x_1} = b^{x_2} \Leftrightarrow x_1 = x_2$

Exponential Growth and Decay:

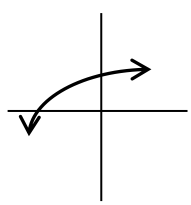
When your growth factor is constant $f(x) = a(1+r)^x$ and $f(x) = a(1-r)^x$.

Compound interest: $A(x) = P(1 + \frac{r}{n})^{nt}$

Continuous compounding interest : $A(x) = Pe^{rt}$

In general terms: $A(t) = A_0(e^{kt})$ where ***k*** is a constant of growth or decay

43) The graph of $y = 2 - a^{x+3}$ for $a > 1$ is best represented by which graph?

A.


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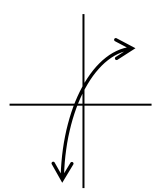

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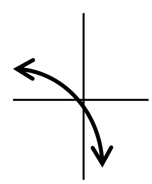

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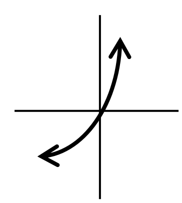


44) Rewrite the expression $\log_5(3x^2y^4)$ into an equivalent expression by expanding.

45) Solve: $\log_6(x+3) + \log_6(x+4) = 1$

46) Solve: $\log x^2 - \log 100 = \log 1$

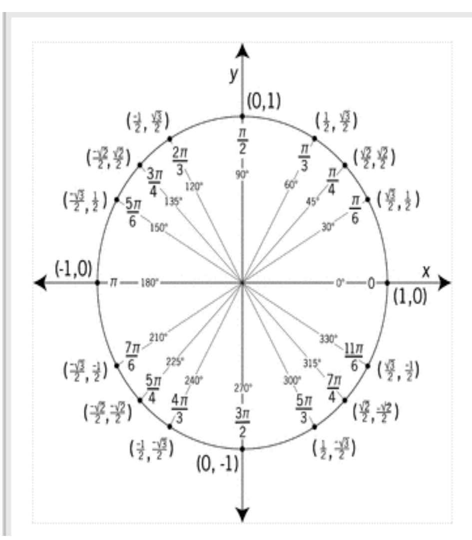
42) The population $\boldsymbol{P}$ of a culture of bacteria where t is the time, in hours, relative to the time at which the population was 1600 is described by the equation ,
$P(t) = 1600e^{0.052t}$

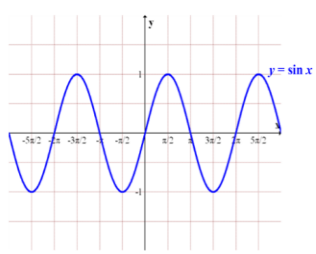
(a) What was the population at $t = 6$ hours?

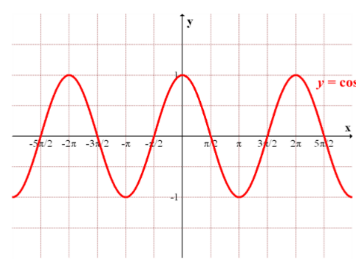
(b) After how many hours will the population reach 8000? Round to the nearest tenth of an hour.

48) Arturo invests \$2700 in a savings account that pay 9% interest, compounded quarterly. If there are no other transactions, when will his balance reach \$4550?

Topic 9: Trigonometry





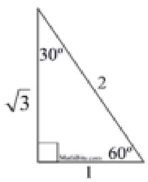


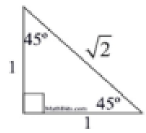
For $y = A\sin(B(x - C)) + D$

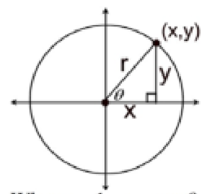
- Amplitude $= |A|$
- Frequency $= B$
- Period $= \dfrac{2\pi}{B}$
- Horizontal Shift $= C$
- Vertical Shift $= D$

Functions:

- $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{y}{r}$
- $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{x}{r}$
- $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{y}{x}$
- $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{r}{y}$
- $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{r}{x}$
- $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{x}{y}$







When $r = 1$, $x = \cos\theta$ and $y = \sin\theta$.

θ degrees	θ radians	sin	cos	tan
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	undefined

Degrees and Radians:

- *Convert Degrees to Radians*: multiply by $\dfrac{\pi}{180}$
- *Convert Radians to Degrees*: multiply by $\dfrac{180}{\pi}$

- Radian measure θ of a central angle of a circle is defined as the ratio of the length of the arc the angle subtends, s, divided by the radius of the circle, r. $\theta = \dfrac{s}{r}$ or $s = \theta r$

Sketch the graphs using the intercepts, amplitude, period, frequency and midline.

49) . $f(x) = 2\sin\theta$

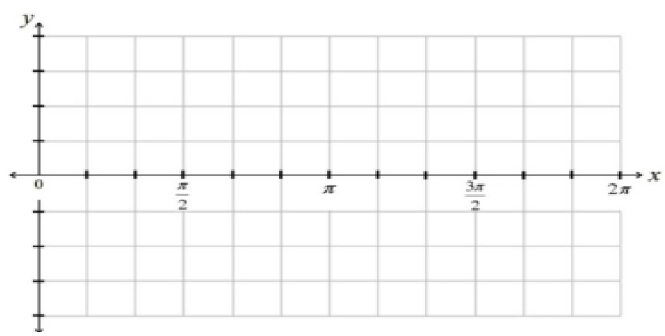
Intercepts:

Amplitude:

Period:

Frequency:

Midline:



50) . $f(x) = 2\cos(2\theta)$

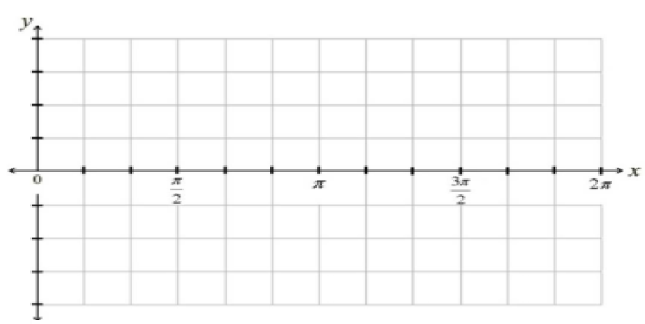
Intercepts:

Amplitude:

Period:

Frequency:

Midline:



51) Find the exact value of each without the use of a calculator.

a) $\sin(3\pi) =$ b) $\cos\left(-\frac{3\pi}{2}\right) =$

c) $\tan\left(-\frac{5\pi}{6}\right) =$ d) $\csc\left(\frac{2\pi}{3}\right) =$

e) $\cot\left(\frac{\pi}{2}\right) =$ f) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

52) Find the perimeter of a 30° slice of cheesecake if the radius of the cheesecake is 8 inches.

53) Two students are 180 feet apart on opposite sides of a telephone pole. The angles of elevation from the students to the top of the pole are 35° and 23°. Find the height of the telephone pole.

54) Find the complement of $\frac{\pi}{14}$.

55) Rewrite 47.10° in radian measure. Round to three decimal places.

56) Rewrite $\frac{4\pi}{25}$ in degree measure. Round to three decimal places.

57) After leaving the runway, a plane's angle of ascent is 16° and its speed is 267 feet per second. How many minutes will it take for the airplane to climb to a height of 12,500 feet? Round answer to two decimal places.

Topic 9B: Trigonometric Identities and Equations

Ratios:

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocals:

- $\csc\theta = \frac{1}{\sin\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$
- $\cot\theta = \frac{1}{\tan\theta}$

Odd/Even:

$f(-x) = f(x) \rightarrow$ EVEN
$f(-x) = -f(x) \rightarrow$ ODD

- cosine is even: $\cos(-x) = \cos(x)$
- sine is odd: $\sin(-x) = -\sin(x)$
- tangent is odd: $\tan(-x) = -\tan(x)$

Pythagorean Identities:

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

Double Angles:

- $\sin 2A = 2\sin A\cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 2A = 1 - 2\sin^2 A$
- $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

Angle Sum/Difference Identities:

- $\sin(A+B) = \sin A\cos B + \cos A\sin B$
- $\cos(A+B) = \cos A\cos B - \sin A\sin B$
- $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$
- $\sin(A-B) = \sin A\cos B - \cos A\sin B$
- $\cos(A-B) = \cos A\cos B + \sin A\sin B$
- $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}.$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha.$$

58) Simplify $(\csc(x) - \tan(x))\sin(x)\cos(x)$

A. $\sin(x) - \cos^2(x)$ B. $\cos(x) - \sin^2(x)$
C. $\sin^2(x) + \cos(x)$ D. $\cos^2(x) - \sin(x)$

59) Solve the equation $2\sin^2(x)\cos(x) = \cos(x)$ algebraically.

60) Find all the exact solutions to $2\sin^2(x) + 3\sin(x) - 2 = 0$ on the interval $[0, 2\pi)$.