

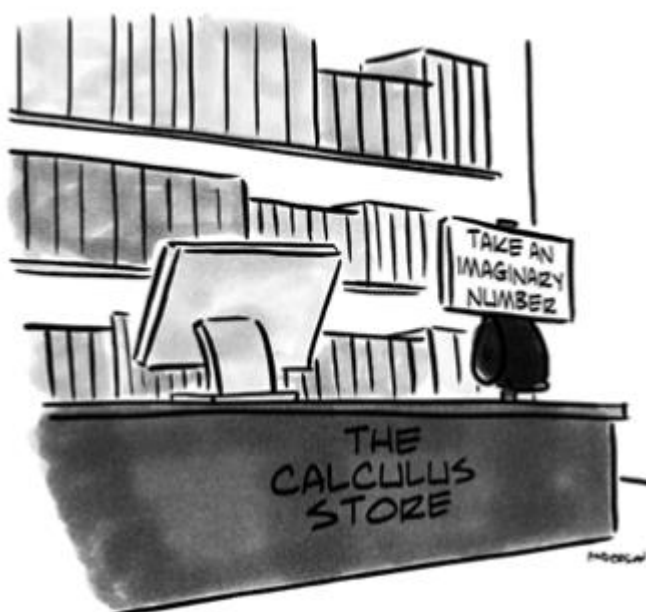


ARCHBISHOP CURLEY

HIGH SCHOOL

AP CALCULUS AB

SUMMER ASSIGNMENT 2023



Name: _____ Date Submitted: _____

AP Calculus AB Summer Packet 2023

Welcome to AP Calculus AB. This will be the toughest class yet in your mathematical careers, but the benefit you will receive by having this experience in high school is immense. Because of the unique nature of this class, it is very important that you are ready to start working on the **first day**. We will NOT be reviewing the material in this packet. The reason for this is to make sure we have adequate time for all of the new material that you did not see this year, and more importantly, to make sure we have completed all of the required material well before the AP exam.

As the instructor of AP Calculus, I have extremely high expectations of students taking this course. Expected is a certain level of independence by anyone taking AP Calculus. Your first opportunity to demonstrate your capabilities and resourcefulness is through this summer work packet which will help you maintain/improve your skills. This packet is a requirement for those entering AP Calculus AB and is **due the first day of class**.

There are certain math skills that have been taught to you over the previous years that are necessary to be successful in calculus. If you do not fully understand the topics in this packet, it is possible that you will get calculus problems wrong in the future not because you do not understand the calculus concept, but because you do not understand the algebra or trigonometry behind it. Don't fake your way through any of these problems because you will need to understand everything in this very well. You may work with someone else while you do this, but copying will not be tolerated. Also, don't wait until the last minute to do everything in the packet because you may run out of time and rush through them. Likewise, do not do all the problems right at the beginning of summer and completely forget how to do all of them by the time school starts again.

If you feel that you need help with some of these topics, the best resource may be the internet. There are great sites where you can just type in the topics to get help. Using [youtube.com](https://www.youtube.com), [khanacademy.org](https://www.khanacademy.org), not only will help you on this packet this summer, but will be referenced often throughout the upcoming school year.

For this packet you must show all of your work on a separate sheet of paper and circle your answer. If I cannot find each question and answer easily I will assume they are not there or incorrect, so make sure your work is neat. Do not rely on a calculator to do all of the work for you. Half of the AP exam does not allow any calculator at all. This packet will be worth a **test grade**.

Additional Requirements:

In addition to this assignment, you will be required to register with our course on MATH-XL. You will have an assignment to do to introduce you to the course and the feature of the online program. Once you have completed the questions in this packet, you will be required to complete a PreRequisite Skills test online on MATH XL. We will be using this assessment tool throughout the course. Your first attempt at the Prerequisite Skills Test will be completed *before the first day of class*.

Textbook:

Required: Finney, Demana, Waits and Kennedy, *Calculus: Graphical, Numerical, Algebraic*, 4th AP® Edition ISBN: 13-9780133178579

Calculator:

Required: **TI nspire CX CAS/TI nspire CAS**. If you have a non-CAS TI nspire or a TI-84+/CE, you may still use these calculators. You can sign out a CAS calculator from me before school ends in June. There are some functions that can only be performed on a CAS. The College Board allows you to use two calculators, if you wish, on the AP exam.

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This packet contains the topics that you have learned in your previous courses that are most important to calculus. Make sure that you are familiar with all of the topics listed below when you enter Calculus.

Algebra Basics

- ☐ You must be able to factor polynomials (GCF, by grouping, trinomials, difference of squares, and sum & difference of cubes)
- ☐ You must know how to add, subtract, multiply, and divide rational expressions.
- ☐ You must know how to work with complex fractions

Function Basics

We use functions almost every day in calculus. It is critical that you are able to identify and do the following:

- ☐ Identify functions from equations, graphs, sets; Use function notation
- ☐ Find the domain and range of a function
- ☐ Identify functions as even, odd or neither. You should also be able to identify what lines of symmetry go with even and odd functions.
- ☐ Perform operations on functions; Compose functions; Find inverses of functions.
- ☐ Graph and write the equations of piecewise functions.
- ☐ Identify discontinuities.
- ☐ Use long and synthetic division and factoring
- ☐ Find the left hand and right hand behavior of a graph.
- ☐ Find the zeros of a polynomial function algebraically and graphically.
- ☐ Find vertical, horizontal, and slant asymptotes and any holes in the graph of a function.

Logarithmic and Exponential Functions

- ☐ Know the properties of exponents and logarithms.
- ☐ Natural logs will be used a lot in Calculus, so be familiar with them.
- ☐ Solve exponential and logarithmic equations

Trigonometry

We use the trig functions and a few of the identities regularly in Calculus. The following are the most important:

- ☐ Calculus is always in radians, never degrees.
- ☐ Know the special right triangles!
- ☐ You need to know, without a calculator, the exact values of \sin , \cos , \tan , \csc , \sec , \cot for the following values: $\pi, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{4}, 2\pi$
- ☐ Be able to identify the graph of sine, cosine, and tangent and the important characteristics
- ☐ The Pythagorean Identities and Double Angle Identities are the most used.
- ☐ Be able to work with arcsin, arccos, and arctan.

Limits

- ☐ Be able to identify the limit of from graphs and tables
- ☐ Be able to identify limits using algebraic methods
- ☐ Be able to find limits at infinity (limits where $x \rightarrow \infty$ or $-\infty$)
- ☐ Be able to find infinite limits (behavior around asymptotes)

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Summer Work Instructions and Due Dates

PART 1: Review Question Packet

You must complete all of the problems in the Review packet. You may consult one another or other resources for material you may have forgotten. This packet will be your first major grade of the first quarter. You should complete this packet before doing the PreTest, by August 29, 2023. In any event, you must submit it on the first day of class.

PART 2: MATH XL registration, introductory assignment and pre-test

From the information provided to you in your folder, use the Math XL code to get registered in our class. This is a resource directly connected to our text and one we will use for section quizzes throughout the course. You **MUST** get registered in the course and complete the introductory assignment which will guide you through the site.

You must **register by August 1, 2023.**

The introductory assignment must be completed by **August 28, 2023**

Once you have completed the Intro assignment, you will take the “Getting Ready for Calculus Pretest”. If you have completed your packet, this pretest should give you a good idea how well you have mastered the prerequisite material.

The Pretest must be completed on or before **September 4, 2023.**

From the results of the summer assignment and the pretest, we will review key concepts the first week of classes and then test on this material.

Your completed packet (with folder), will be submitted on the first day of class.

Do not leave the work on this packet until the last minute. Do some problems each week. The pretest could take some time to complete. You may do it in parts as well. You may use your calculator unless otherwise indicated.

I will see you all in August. Happy Calculus-ing!

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Answer every question. Show all work in clear, legible and organized manner. Clearly mark all answers. Label all diagrams and graphs appropriately. Use separate pages for work when necessary.

I. Exponents and Radicals:

Simplify:

1) $g^5 \cdot g^{11}$ _____

2) $(b^6)^3$ _____

3) w^{-7} _____

4) $\frac{y^{12}}{y^8}$ _____

5) $(3x^7)(-5x^{-3})$ _____

6) $(-4a^{-5}b^0c)^2$ _____

7) $\frac{-15x^7y^{-2}}{25x^{-9}y^5}$ _____

8) $\left(\frac{4x^9}{12x^4}\right)^3$ _____

Express the following in simplest radical form.

9) $\sqrt{50}$

10) $\sqrt{24}$

11) $\sqrt{192}$

12) $\sqrt{169}$

13) Simplify $\frac{\sqrt[7]{x^9}}{\sqrt[5]{x^6}}$. Express your answer using a single radical.

II. Factoring Polynomials

Factor completely:

14) $5x^2 - 32x - 21$

15) $4x^2 + 20x + 9$

16) $15x^3 - 25x^2 + 75x - 125$

17) $x^2 + 15x + 56$

18) $28x^3 - 7x$

19) $216x^3 + 1$

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Complex Fractions and Rational Expressions

Simplify:

$$20) \quad \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} =$$

$$21) \quad \frac{1 - \frac{1}{x^3}}{3 + \frac{1}{x^2}} =$$

$$22) \quad \frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}} =$$

$$23) \quad \frac{5 + \frac{1}{n} - \frac{6}{n^2}}{\frac{2}{n} - \frac{2}{n^2}} =$$

Function Composition and Inverses

$$24) \quad \text{Let } f(x) = \sqrt{x-3} \text{ and } g(x) = x^2 + 1. \text{ Compute } (g \circ f)(x),$$

$$25) \quad \text{Find } f(g(x)) \text{ if } f(x) = 2x^2 \text{ and } g(x) = x + 4$$

$$26) \quad \text{Let } f(x) = \frac{3x+7}{x-2}. \text{ Find } f^{-1}(x), \text{ the inverse of } f(x).$$

$$27) \quad \text{Find } f^{-1}(x) \text{ if } f(x) = \sqrt{x^2 + 16}$$

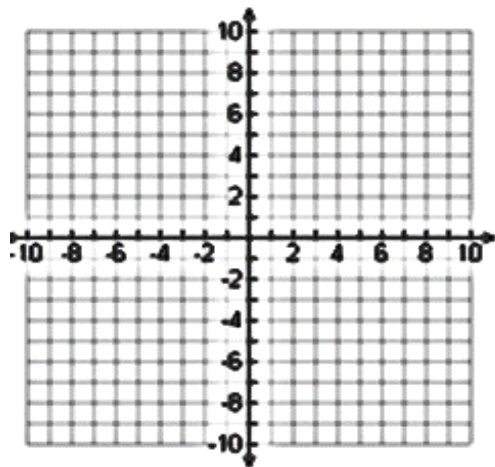
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Linear and Quadratic Functions

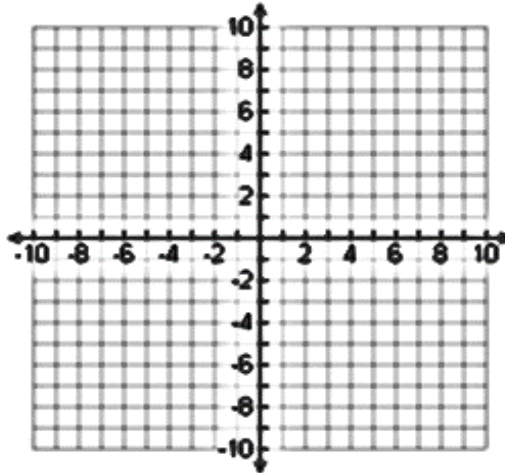
28) Write an equation of a line with slope 3 and y-intercept 5.

29) Use point-slope form of a linear equation to find an equation of the line passing through the point (6, 5) with a slope of $2/3$.

30) Find an equation of a line passing through the points (-3, 6) and (1, 2).
Graph it below.



31) Find an equation of a line with an x-intercept of (2, 0) and a y-intercept of (0, 3).
Graph it below.



32) Find an equation of a line with an x-intercept of (2, 0) and a y-intercept of (0, 3).

33) Solve the inequality: $x^2 - x - 12 > 0$.

34) Find an equation for the parabola whose vertex is (2, -5) and passes through (4, 7). Express your answer in the vertex form for a quadratic.

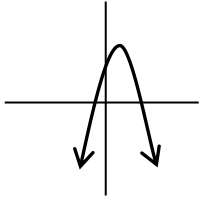
35) Transform $y = -3x^2 - 24x + 11$ to vertex form by completing the square.

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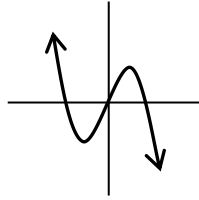
Polynomial and Rational Functions

36) Which of the following could represent a complete graph of $f(x) = ax - x^3$, where a is a real number?

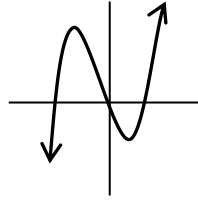
A.



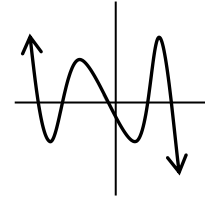
B.



C.



D.



37) Find a degree 3 polynomial with zeros -2, 1, and 5 and going through the point (0, -3).

38) Use polynomial long division to rewrite the expression $\frac{x^3 + 7x^2 + 14x - 8}{x - 4}$

39) Use a graphing calculator to approximate all of the function's real zeros. Round your results to four decimal places. $f(x) = 3x^6 - 5x^5 - 4x^3 + x^2 + x + 1$

40) Give that $f(x) = \frac{\sqrt{x+5}}{x+2}$. Find the asymptotes and the domain of the function.

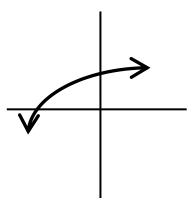
Domain: _____ Vertical Asymptote(s): _____ Horizontal Asymptote(s): _____

41) Factor to solve the inequality. Write your answer in interval notation. $0 \leq \frac{x^3 - 64}{x - 3}$

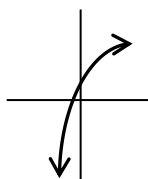
III. Exponential and Logarithmic Functions

42) The graph of $y = 2 - a^{x+3}$ for $a > 1$ is best represented by which graph?

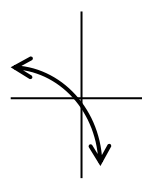
A.



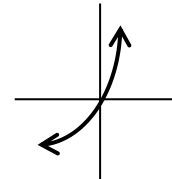
B.



C.



D.



43) Rewrite the expression $\log_5(3x^2y^4)$ into an equivalent expression by expanding.

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44) Solve: $\log_6(x+3) + \log_6(x+4) = 1$

45) Simplify: $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$

46) Solve: $\log x^2 - \log 100 = \log 1$

47) The number of elk after t years in a state park is modeled by the function $P(t) = \frac{1216}{1 + 75e^{-0.03t}}$.

a. What was the initial population of elk?

b. When will the number of elk be 750?

IV. Trigonometric Functions

Sketch the graphs using the intercepts, amplitude, period, frequency and midline.

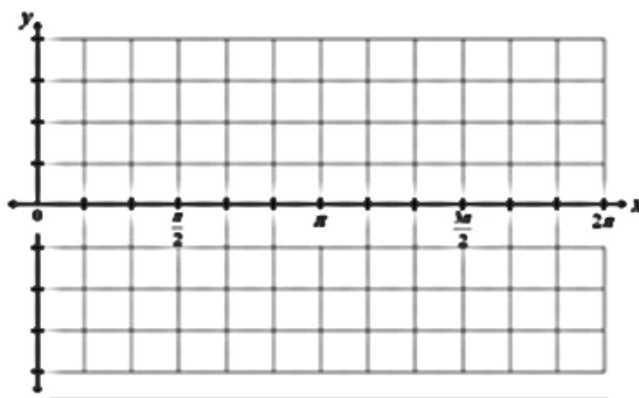
48) $f(x) = 2 \sin \theta + 1$

Intercepts:

Amplitude:

Period:

Midline:



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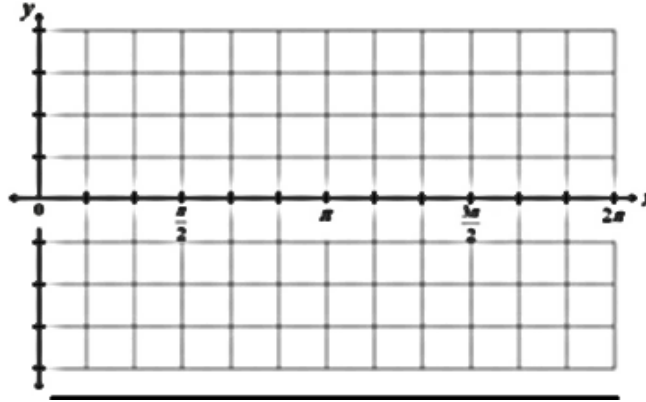
49) $f(x) = 2\cos(2\theta)$

Intercepts:

Amplitude:

Period:

Midline:



50) Given $\sin 30^\circ = \frac{1}{2}$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$, determine the exact value of $\csc 30^\circ$

A) $\csc 30^\circ = \frac{\sqrt{3}}{3}$

B) $\csc 30^\circ = \frac{\sqrt{2}}{2}$

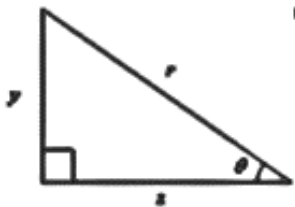
C) $\csc 30^\circ = \sqrt{3}$

D) $\csc 30^\circ = 2$

E) undefined

51) If $\sin \theta = \frac{\sqrt{3}}{2}$, find the value of $\sec \theta$ in degrees ($0 < \theta < 90^\circ$) without the aid of a calculator.

52) Using the figure below, if $\theta = 26^\circ$ and $y = 8$, determine the value of x .



V. Trigonometric Identities

53) Simplify $(\csc(x) - \tan(x))\sin(x)\cos(x)$

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54) Solve the equation $2\sin^2(x)\cos(x) = \cos(x)$ algebraically.

55) Find all the exact solutions to $2\sin^2(x) + 3\sin(x) - 2 = 0$ on the interval $[0, 2\pi)$.

56) Solve for x : $3\sin^2 x = \cos^2 x$ for interval $0 \leq x \leq 2\pi$

57) Solve for x : $\cos^2 x - \sin^2 x = \sin x$ for interval $-\pi \leq x \leq \pi$

VI. Limits and Continuity (for students coming from Algebra 2 Honors or PreCalculus, this section is optional – Q 58-67) You are certainly encouraged to try these problems using the math you know or by researching from your textbook. We will be covering this material in Unit 1.

58)

$$\lim_{x \rightarrow -2} \sqrt{x^4 + 3x + 6} =$$

59)

$$\lim_{x \rightarrow 3} \sqrt{25 - x^2} =$$

60)

$$\lim_{x \rightarrow 5} \frac{x^2 - 6x + 5}{x - 5} =$$

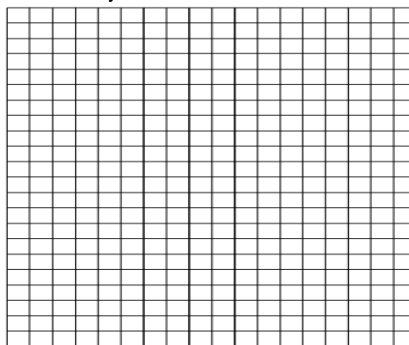
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61) $\lim_{x \rightarrow \infty} \frac{3x^3}{x^2 - 1} =$

62) $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + 3}}{5 - 2x} =$

63) Sketch the graph of $f(x)$ and then describe the continuity of the function

$$f(x) = \begin{cases} x + 2, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$$



64) Let $\lim_{x \rightarrow -8} f(x) = 4$ and $\lim_{x \rightarrow -8} g(x) = 7$ Find $\lim_{x \rightarrow -8} [f(x) \cdot g(x)]$

65) $\lim_{x \rightarrow \infty} \frac{x^2 + 8x + 3}{x^3 + 4x^2 + 5} =$

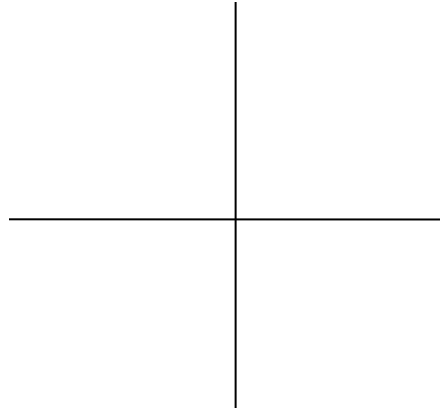
66) $\lim_{x \rightarrow 5} \frac{\sqrt{x+4} - 3}{x-5} =$

67) Determine the value of k so that the function $g(x) = \begin{cases} x^2 + 2, & x > 2 \\ kx + 4, & x \leq 2 \end{cases}$ is continuous.

VII. Miscellaneous Problems:

68) Graph the piecewise function.

$$f(x) = \begin{cases} -x^2 & -2 \leq x < 1 \\ -2 & x = 1 \\ 3x + 5 & 1 < x \leq 3 \end{cases}$$



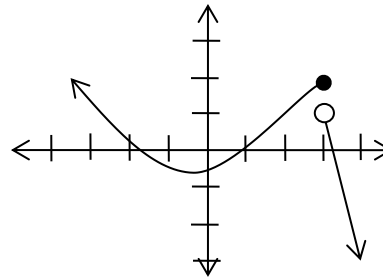
69) For the function $f(x)$ graphed answer the following

A. $f(3)$

B. $f(x) = 0$

C. $f(0)$

D. $f(x) = 1$



70) Given : $y = \frac{2(x-3)(x+2)}{(x-3)(x+3)}$

Identify:

a. any holes _____

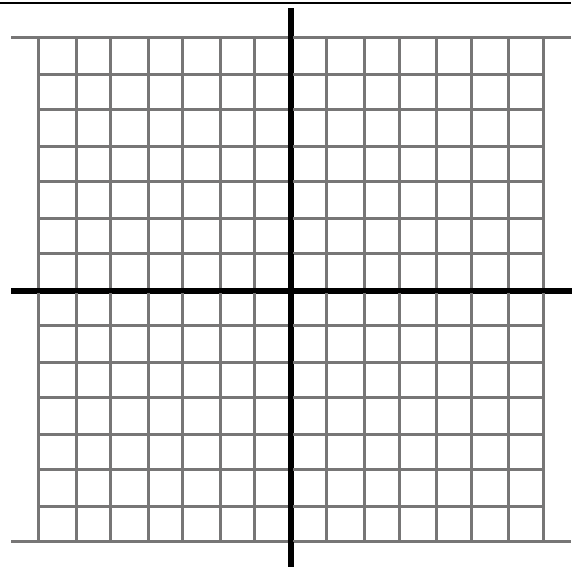
b. any vertical asymptotes _____

c. any horizontal asymptotes _____

d. x - intercepts _____

e. y - intercepts _____

f. Graph.

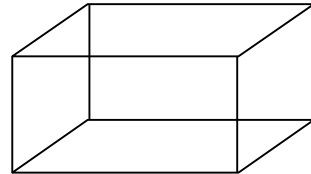


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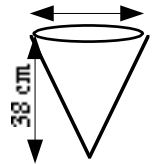
- 71) Simplify the expression as much as possible. $10x^2 \cdot \frac{1}{5}(3x^2 - 4)^{-4/5} 6x + (3x^2 - 4)^{1/5} \cdot 20x$

- 72) A **closed** box with a square base of side (x) and height (y) has a surface area of 100 sq. ft.

Express the volume of the box as a function of x .
 $V(x) =$



- 73) Water is poured into a conical container with a diameter of 20 cm and a height of 38 cm. If the volume of a cone is one-third the volume of a cylinder, write an equation for the volume of water as a function of the height of the water in the container.



- 74) Simplify the expression as much as possible. $\frac{\frac{1}{\sqrt{3(x+h)}} - \frac{1}{\sqrt{3x}}}{h}$

- 75) Two students are 180 feet apart on opposite sides of a telephone pole. The angles of elevation from the students to the top of the pole are 35° and 23° . Find the height of the telephone pole.