AP Calculus BC Summer Assignment

Complete all problems. Show all of your work. Make sure your work is organized and legible. Attach extra pages where necessary. Do not use a calculator unless specifically stated.

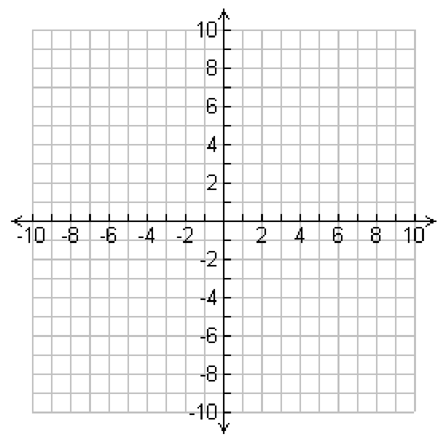
Part I: First, let's whet your appetite with a little Precalc!

1) For what value of *k* are the two lines $2x+ky=3$ and $x+y=1$

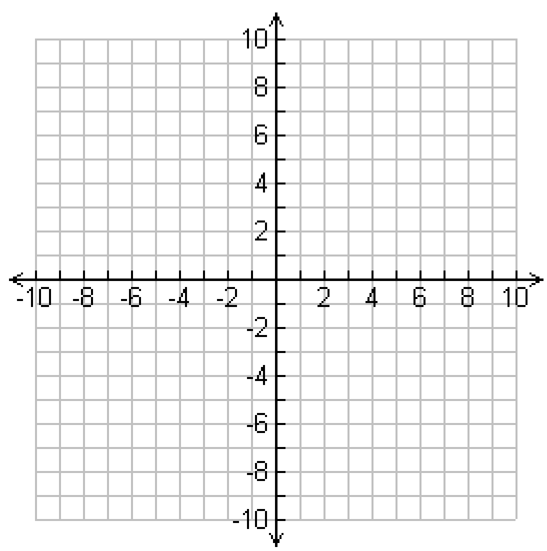
(a) parallel? (b) perpendicular?

2) Graph the function shown below. Also indicate any key points and state the domain and range.

$$f(x)=\begin{cases} 4-x^2, & x<1 \\ \frac{3}{2}x+\frac{3}{2}, & 1\le x\le 3 \\ x+3, & x>3 \end{cases}$$



3) Graph the function $y=3e^{-x}-2$ and indicate asymptote(s). State its domain, range, and intercepts.



<u>Part II: Unlimited and Continuous!</u>

For #1-4 below, find the limits, if they exist.

1) $\lim_{x \to 4} \frac{2x^3 - 7x^2 - 4x}{x - 4}$

2) $\lim_{x \to 9} \frac{\sqrt{x} - 3}{9 - x}$

3) $\lim_{x \to 1} \frac{x^2 - 2x - 5}{x + 1}$

4) $\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$

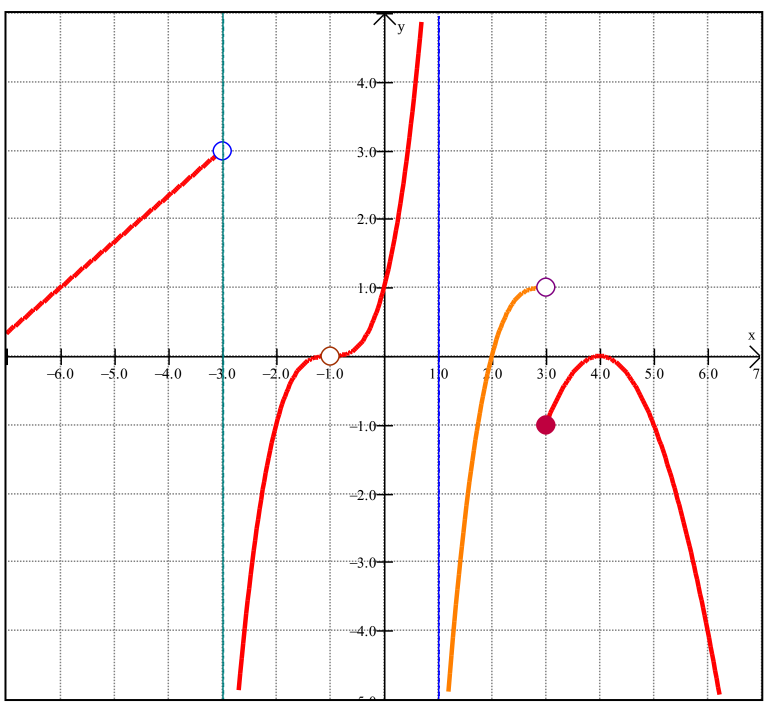
For #5-7, explain why each function is discontinuous and determine if the discontinuity is removable or nonremovable.

5) $g(x) = \begin{cases} 2x - 3, & x < 3 \\ -x + 5, & x \geq 3 \end{cases}$

6) $b(x) = \frac{x(3x+1)}{3x^2 - 5x - 2}$

7) $h(x) = \frac{\sqrt{x^2 - 10x + 25}}{x - 5}$

For #8-13, determine if the following limits exist, based on the graph below of p(x). If the limits exist, state their value. Note that x = -3 and x = 1 are vertical asymptotes.



8) $\lim_{x \to 1^-} p(x)$

9) $\lim_{x \to -3^-} p(x)$

10) $\lim_{x \to 2} p(x)$

11) $\lim_{x \to 3^-} p(x)$

12) $\lim_{x \to 3^+} p(x)$

13) $\lim_{x \to -1} p(x)$

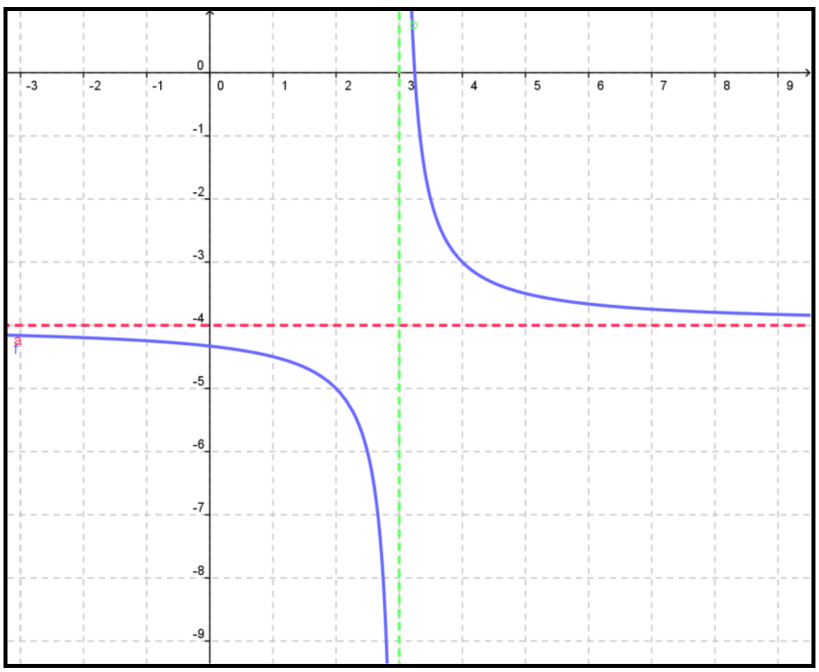
14) Consider the function $f(x)=\begin{cases} x^2+kx & x\le 5 \\ 5\sin\left(\frac{\pi}{2}x\right) & x>5 \end{cases}$,

In order for the function to be continuous at x = 5, the value of *k* must be

15) Consider the function $f(x)=\begin{cases} \frac{\sin x}{x} & x\ne 0 \\ k & x=0 \end{cases}$.

In order for the function to be continuous at x = 0, the value of *k* must be

Use the graph of f(x), shown below, to answer #16-18.



16) For what value of a is $\lim_{x\to a} f(x)$ nonexistent?

17) $\lim_{x\to\infty} f(x)=$__________ 18) $\lim_{x\to-\infty} f(x)=$__________

Part III: Designated Deriving

1) $\lim_{h\to 0} = \dfrac{\tan^{-1}(1+h)-\tan^{-1}(1)}{h} =$

2) $\lim_{h\to 0} = \dfrac{\sec(\pi+h)-\sec(\pi)}{h} =$

For #3-8, find the derivative.

3) $y = \ln(1+e^x)$

4) $y = \csc(1+\sqrt{x})$

5) $y = (\tan^2 x)(3\pi x - e^{2x})$

6) $y = \sqrt[7]{x^3 - 4x^2}$

7) $f(x) = (x+1)e^{3x}$

8) $f(x) = \dfrac{e^{x/2}}{\sqrt{x}}$

9) If $xy^2 - y^3 = x^2 - 5$, then $\dfrac{dy}{dx} =$

10) The distance of a particle from its initial position is given by $s(t) = t - 5 + \dfrac{9}{(t+1)}$, where *s* is feet and *t* is minutes. Find the velocity at *t* = 1 minute in appropriate units.

Use the table below for #11-12.

X	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	4	2	5	½
3	7	-4	$\frac{3}{2}$	-1

11) The value of $\dfrac{d}{dx}(f \cdot g)$ at x = 3 is

12) The value of $\dfrac{d}{dx}\left(\dfrac{f}{g}\right)$ at x = 1 is

In #13-14, use the table below to find the value of the first derivative of the given functions for the given value of x.

X	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	3	2	0	¾
2	7	-4	$\frac{1}{3}$	-1

13) $[f(x)]^2$ at x = 2 is

14) $f(g(x))$ at x = 1 is

15) Let f be the function defined by $f(x)=\dfrac{x+\sin x}{\cos x}$ *for* $-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of *f* at the point $(0, f(0))$.

Part IV: Derived and Applied!

For #1-3, find all critical values, intervals of increasing and decreasing, any local extrema, points of inflection, and all intervals where the graph is concave up and concave down.

1) $f(x)=\dfrac{5-4x+4x^2-x^3}{x-2}$

2) $y=3x^3-2x^2+6x-2$

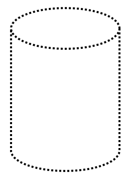
3) $f'(x)=5x^3-15x+7$

4) The graph of the function $y=x^5-x^2+\sin x$ changes concavity at x =

5) Find the equation of the line tangent to the function $y=\sqrt[4]{x^7}$ at $x=16$.

6) For what value of x is the slope of the tangent line to $y=x^7+\dfrac{3}{x}$ undefined?

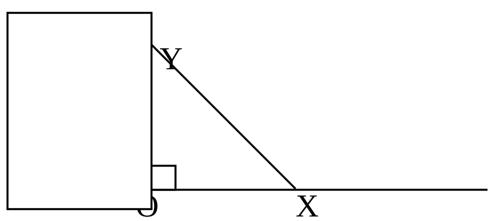
7)



The balloon shown is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder with radius *r* and height *h* is $\pi r^2 h$, and the volume of a sphere with radius *r* is $\frac{4}{3}\pi r^3$.)

(a) At this instant, what is the height of the cylinder?
(b) At this instant, how fast is the height of the cylinder increasing?

8)



A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at a constant rate of ½ foot per second.

(a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
(b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

Part V: Integral to Your Success!

1) $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}} dx$

2) $\int_{-\pi/6}^{\pi/6} \sec^2 x dx$

3) $\frac{d}{dx} \int_1^x \sqrt[4]{t}\, dt$

4) $\frac{d}{dx} \int_{\sin(4x)}^{0} e^t dt$

5) $\int \frac{x^3}{\sqrt{1 + x^4}} dx$

6) $\int \frac{\csc^2 x}{\cot^3 x} dx$

7) $\int \sqrt{\tan x} \sec^2 x dx$

8) What are all the values of *k* for which $\int_2^k x^5 dx = 0$?

9) What is the average value of $y = x^3\sqrt{x^4 + 9}$ on the interval [0, 2]?

10) If $\int_a^b g(x)dx = 4a + b$, then $\int_a^b [g(x) + 7]dx =$

11) The function *f* is continuous on the closed interval [1, 9] and has the values given in the table. Using the subintervals [1, 3], [3, 6], and [6, 9], what is the value of the trapezoidal approximation of $\int_1^9 f(x)dx$?

x	1	3	6	9
f(x)	15	25	40	30

12) The table below provides data points for the continuous function $y = h(x)$.

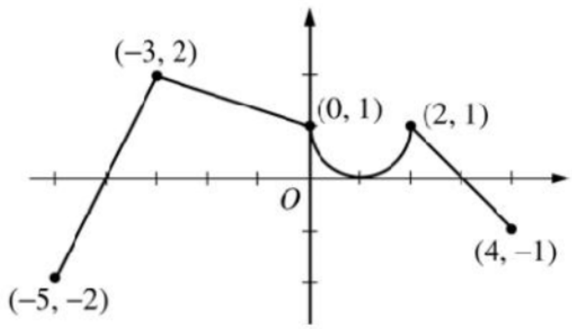
x	0	2	4	6	8	10
h(x)	9	25	30	16	25	32

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of $y = h(x)$ on the interval [0, 10].

13) A particle moves along the x-axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is -9, and its position is -27.

(a) Find $v(t)$, the velocity of the particle at any time $t \geq 0$.
(b) For what values of $t \geq 0$ is the particle moving to the right?
(c) Find $x(t)$, the position of the particle at any time $t \geq 0$.

Part VI: Fundamental Theorem



Graph of f

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.